Problema n. 3

a)

$$[K_1]\cdot[l^0]=[m\cdot l\cdot t^{-2}] \quad\Rightarrow\quad [K_1]=[m\cdot l\cdot t^{-2}]$$

$$[K_2]\cdot[l^1]=[m\cdot l\cdot t^{-2}] \quad\Rightarrow\quad [K_2]=[m\cdot t^{-2}]$$

$$[K_3]\cdot[l^2]=[m\cdot l\cdot t^{-2}] \quad\Rightarrow\quad [K_3]=[m\cdot l^{-1}\cdot t^{-2}]$$

b)

$$\vec{\nabla}\wedge\vec{F}=\vec{0} \quad\Rightarrow$$

$$\frac{\partial F_x}{\partial z}-\frac{\partial F_z}{\partial x}=0,\quad \frac{\partial F_y}{\partial z}-\frac{\partial F_z}{\partial y}=0,\quad \frac{\partial F_x}{\partial y}-\frac{\partial F_y}{\partial x}=0$$

c)

$$\vec{F}=\vec{\nabla}U$$

$$F_x=\frac{\partial U}{\partial x},\quad F_y=\frac{\partial U}{\partial y},\quad F_z=\frac{\partial U}{\partial z}$$

$$U(x,y,z)=\int\vec{F}\cdot d\vec{r}=\int\left(K_1\hat{i}-K_2y\hat{j}+K_3z^2\hat{k}\right)\left(dx\hat{i}+dy\hat{j}+dz\hat{k}\right)=\int\left(K_1dx-K_2ydy+K_3z^2dz\right)$$

$$U(x,y,z)=K_1x-\frac{1}{2}K_2y^2+\frac{1}{3}K_3z^3$$

d)

$$U(O)=U(0,0,0)=0$$

$$U(B)=U(0,3,0)=-9J$$

$$L_{OB}=U(B)-U(O)=-9J$$

Problema n. 4

Forze che agiscono sullo *yo-yo*:

$$\vec{F}_R = \vec{F}_A + \vec{F}_T + \vec{F}_C$$

$$\vec{F}_A = \vec{P} + \vec{T}$$

$$\vec{F}_T = -m\vec{a}_0$$

$$\vec{F}_C = 0$$

Equazioni cardinali della dinamica:

$$\begin{cases} P - T - ma_0 = ma \\ TR = I\dot{\omega} \end{cases}$$

$$\dot{\omega} = \frac{\|\vec{a}\|}{R} = \frac{a}{R}$$

$$TR = I\frac{a}{R}$$

$$T = \frac{1}{2}mR^2\frac{a}{R} = \frac{1}{2}ma$$

$$\not{m}g - \frac{1}{2}\not{m}a - \not{m}a_0 = \not{m}a$$

$$\frac{3}{2}a = g - a_0 \quad \Rightarrow \quad a = \frac{2}{3}(g - a_0) \quad \Rightarrow \quad a = \frac{2}{3}(9.8 - 1.5) = 5.5\overline{3}\,ms^{-2}$$

Problema n. 5

a)

Conservazione della componente orizzontale della quantità di moto:

$$Q_i = MV_i + mv_i = 0$$

$$Q_f = MV_f + mv_f = MV_A + mv_A$$

$$\Delta Q = Q_f - Q_i = 0 \quad \Rightarrow \quad Q_f = Q_i$$

$$MV_A + mv_A = 0 \quad \Rightarrow \quad v_A = -\frac{M}{m}V_A$$

Conservazione dell'energia meccanica:

$$T_i = \frac{1}{2}MV_i^2 + \frac{1}{2}mv_i^2 = 0$$

$$U_i = Mgh_M + mgh_m = Mg \cdot 0 + mgh = mgh$$

$$T_f = \frac{1}{2}MV_f^2 + \frac{1}{2}mv_f^2 = \frac{1}{2}MV_A^2 + \frac{1}{2}mv_A^2$$

$$U_f = Mg \cdot 0 + mg \cdot 0 = 0$$

$$\Delta E = E_f - E_i = 0 \quad \Rightarrow \quad E_f = E_i \quad \Rightarrow \quad T_f + U_f = T_i + U_i$$

$$\frac{1}{2}MV_A^2 + \frac{1}{2}mv_A^2 = mgh$$

$$\frac{1}{2}MV_A^2+\frac{1}{2}m\frac{M^2}{m^2}V_A^2=mgh$$

$$\frac{1}{2}M\left(1+\frac{M}{m}\right)V_A^2=mgh$$

$$V_A^2=\frac{2m^2gh}{M(M+m)}\quad\Rightarrow\quad V_A=\sqrt{\frac{2m^2gh}{M(M+m)}}=\sqrt{\frac{2\cdot 1^2\cdot 9.8\cdot 4}{3(3+1)}}=2.56ms^{-1}$$

b) $$v_A=-\frac{M}{m}V_A=-\frac{3}{1}2.56=7.67ms^{-1}$$

$$\frac{1}{2}mv_U^2=\frac{1}{2}mv_A^2-L_{T_r}$$

$$\vec{T}_r=-\mu\vec{N}=-\mu mg$$

$$L_{T_r}=\mu mg\int_0^d dx=\mu mgx\Big|_0^d=\mu mgd$$

$$\frac{1}{2}\cancel{m}v_U^2=\frac{1}{2}\cancel{m}v_A^2-\mu\cancel{m}gd$$

$$v_U^2=v_A^2-2\mu gd$$

$$v_U=\sqrt{v_A^2-2\mu gd}=\sqrt{58.80-2\cdot 0.5\cdot 9.8\cdot 5}=\sqrt{9.80}=3.13ms^{-1}$$

$$\frac{1}{2}\cancel{m}v_U^2=\cancel{m}gh_M$$

$$h_M=\frac{v_U^2}{2g}=\frac{9.80}{2\cdot 9.8}=0.5m$$

c)

$$t = t_d + t_p = t_d + \frac{T_p}{4}$$

$$\mu g = a_{media} = \frac{v_A - v_U}{t_d}$$

$$t_d = \frac{v_A - v_U}{\mu g} = \frac{7.67 - 3.13}{0.5 \cdot 9.8} = 0.93s$$

$$T_p = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{8}{9.8}} = 5.68\,s$$

$$t = 0.93 + \frac{5.68}{4} = 2.35s$$