. 1

$$|\vec{v}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(\pi-\theta) \Rightarrow |v| = \sqrt{|a|^2+|b|^2+2|a||b|\cos\theta}$$

$$|\vec{d}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(\theta) =$$

$$= |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta \Rightarrow |d| = \sqrt{|a|^2+|b|^2-2|a||b|\cos\theta}$$

s. 2

e componente del vettore $\vec{b}$ proiettato sulla
rezione del vettore $\vec{a}$ è dato da: $\frac{\vec{a}\cdot\vec{b}}{|a|^2}\cdot\vec{a}$

. $\vec{a}\cdot\vec{b} = (4\hat{j} - 3\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k}) = 1$

$|\vec{a}|^2 = (4\hat{j}-3\hat{k})\cdot(4\hat{j}-3\hat{k}) = 16+9 = 25$

$\vec{p}_1 = \left(\vec{b}\cdot\frac{\vec{a}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|} = \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\cdot\vec{a} = \frac{1}{25}(4\hat{j}-3\hat{k})$

2. $\vec{a}\cdot\vec{b} = (-2\hat{i}+3\hat{j}-\hat{k})\cdot(4\hat{i}-2\hat{j}) =$

$= -2\cdot 4 - 3\cdot 2 - 1\cdot 0 = -14$

$|\vec{a}| = (-2\hat{i}+3\hat{j}-\hat{k})\cdot(-2\hat{i}+3\hat{j}-\hat{k}) =$

$= (-2)^2 + (3)^2 + 1^2 = 14$

$\vec{p}_2 = -(-2\hat{i}+3j-\hat{k}) = 2i - 3j + \hat{k}$

). $\vec{a}\cdot\vec{b} = (8,2,-3)\cdot(1,0,-1) = 8 + 2\cdot 0 + 3\cdot 1 = 11$

Es. 3

$\vec{S'} = \vec{OA} + \vec{AB}$, per il teorema di pitagora

$S'^2 = |OA|^2 + |AB|^2 = 5^2 + 8^2$

$\vec{S} = \vec{S'} + \vec{BC}$, dove per pitagora risulta

$|\vec{S}|^2 = |\vec{S'}|^2 + |\vec{BC}|^2 = 5^2 + 8^2 + 4^2 \Rightarrow |\vec{S}| = 10,25$

Es. 4

$\vec{f} = \vec{f}_1 + \vec{f}_2 + \vec{f}_3 + \vec{f}_4 =$

$= (5\hat{i} - \hat{j} + 4\hat{k}) + (-2\hat{j} + 3\hat{k}) + (3\hat{i} - 2\hat{j} + 5\hat{k}) +$

$+ (-\hat{i} + 7\hat{j} - 3\hat{k}) =$

$= (5+3-1)\hat{i} + (-1-2-2+7)\hat{j} + (4+3+5-3)\hat{k} =$

$= 7\hat{i} + 2\hat{j} + 9\hat{k} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$

$$\begin{cases} V_x = r\sin\theta\cos\varphi \\ V_y = r\sin\theta\sin\varphi \\ V_z = r\cos\theta \end{cases}$$

$r = \sqrt{V_x^2 + V_y^2 + V_z^2}$

s. 5

$A = (\overline{BE} \cdot \overline{AD}) \cdot \overline{OE}$

$\overline{BE} = |\vec{v_3}| \sin\theta \quad \overline{AD} = |\vec{v_2}| \quad \overline{OE} = |\vec{v_1}|$

$\overline{BE} \cdot \overline{AD} = \vec{v_3} \wedge \vec{v_2} \Rightarrow A = |(\vec{v_3} \wedge \vec{v_2}) \cdot \vec{v_1}|$

$$\vec{v_3} \wedge v_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 6 & 6 \\ 0 & 9 & 0 \end{vmatrix} = 54\hat{j}$$

$$A = |(\vec{v_3} \wedge \vec{v_2}) \cdot \vec{v_1}| = |(54\hat{j}) \cdot (-6\hat{i})| = 324$$

s. 6

$A = \frac{1}{2} \overline{AH} \cdot \overrightarrow{OB}$

$\overline{AH} = \overline{OA} \sin\theta \quad , \quad \overrightarrow{OB} = |\vec{v_0}| \quad , \quad \overline{OA} = |\vec{r_0}|$

$A = \frac{1}{2}|(\vec{r_0} \wedge \vec{v_0})|$. Nel nostro caso

$$A = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 7 & 0 \\ 4 & -3 & -7 \end{vmatrix} = |(49)\hat{i} - (-14)\hat{j} + (-6-28)\hat{k}| = |49\hat{i} + 14\hat{j} - 34\hat{k}|$$

$$A = \sqrt{49^2 + 14^2 + (-34)^2} = 61,26$$

Es. 7 Cartesiane - polari

$v_{r_1} = \sqrt{2^2 + 7^2} = 7,28$

$v_{\theta_1} = \cos^{-1}\left(\frac{0}{\sqrt{2^2+7^2}}\right) = 0$

$v_{\varphi_1} = tg^{-1}\left(\frac{7}{2}\right) = 1,29$ rad

$v_{r_2} = \sqrt{4^2 + 9^2 + (-7)^2} = 8,6$

$v_{\theta_2} = \cos^{-1}\left(\frac{-7}{8,6}\right) = 2,52$ rad

$v_{\varphi_2} = tg^{-1}\left(\frac{3}{4}\right) = 0,64$ rad

Cartesiane - Cilindriche

$v_{r_1} = \sqrt{2^2+7^2} = 7,28$

$v_{z_1} = 0$

$v_{\varphi_1} = tg^{-1}\left(\frac{7}{2}\right) = 1,29$ rad

$v_{r_2} = \sqrt{4^2+3^2} = 5$

$v_{z_2} = -7$

$v_{\varphi_2} = tg^{-1}\left(\frac{3}{4}\right) = 0,64$ rad.

s. 8

Esprimiamo $\vec{e}$ in coordinate cartesiane:

$$\vec{e} = \left(\frac{1}{1+ct}\cos(g\cdot t),\ \frac{1}{1+ct}\sin(gt)\right)$$

deriviamo rispetto al tempo:

$$\frac{d\vec{e}}{dt} = \left(\frac{-c}{(1+ct)^2}\cos(g\cdot t) - \frac{g}{1+ct}\sin(gt),\right.$$

$$\left.\frac{-c}{(1+ct)^2}\sin(gt) + \frac{g}{(1+ct)}\cos(gt)\right)$$

$$|\vec{e}| = \sqrt{p^2+u^2}$$

$$p^2 = \frac{c^2}{(1+ct)^4}\cos^2(gt) + \frac{g^2}{(1+ct)^2}\sin^2(gt) +$$

$$+ \frac{2gc}{\ldots}\cos(gt)\sin(gt)$$

$$u^2 = \frac{c^2}{(1+ct)^4}\sin^2(gt) + \frac{g^2}{(1+ct)^2}\cos^2(gt) +$$

$$- \frac{2gc}{(1+ct)^3}\sin(gt)\cos(gt)$$

$$|\vec{e}| = \sqrt{\frac{c^2}{(1+ct)^4} + \frac{g^2}{(1+ct)^2}} = \frac{1}{(1+ct)^2}\sqrt{c^2+g^2(1+ct)^2}$$

$$|\vec{e}|_{t=3s} \simeq 0.715$$

$$\cos\theta = \frac{\vec{e}\cdot\hat{i}}{||e||} = \frac{-c\cos(gt) - \frac{g}{(1+ct)}\sin gt}{\frac{1}{(1+ct)^2}\sqrt{c^2+g^2(1+ct)^2}}$$

$$- \frac{c(1+ct)^2\cos(gt) + (1+ct)\sin(gt)}{\sqrt{c^2+g^2(1+ct)^2}} \simeq 2.34 \text{ rad}$$