

Esercizio :

se $\vec{a} = 1.8\hat{i} + 5.4\hat{j} + 3\hat{k}$ e $\vec{b} = \hat{i} + 4.2\hat{j} + 3.5\hat{k}$

determinare l'espressione in coordinate cartesiane del vettore $\vec{c} = \vec{a} \times \vec{b}$

$$\vec{c} = \vec{a} \times \vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) \times (b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$$

$$\vec{c} = a_x\hat{i} \times b_x\hat{i} + a_x\hat{i} \times b_y\hat{j} + a_x\hat{i} \times b_z\hat{k} +$$

$$+ a_y\hat{j} \times b_x\hat{i} + \dots + a_z\hat{k} \times b_z\hat{k}$$

$$= a_x b_x (\hat{i} \times \hat{i}) + a_x b_y (\hat{i} \times \hat{j}) + a_x b_z (\hat{i} \times \hat{k}) +$$

$$+ a_y b_x (\hat{j} \times \hat{i}) + \dots + a_z b_z (\hat{k} \times \hat{k})$$

$$\text{ma : } \hat{i} \times \hat{i} = 0 \quad \hat{j} \times \hat{j} = 0 \quad \text{e} \quad \hat{k} \times \hat{k} = 0$$

$$\hat{i} \times \hat{j} = \hat{k} \quad \hat{j} \times \hat{k} = \hat{i} \quad \text{e} \quad \hat{k} \times \hat{i} = \hat{j}$$

in conclusione:

$$\vec{c} = (a_y b_z - a_z b_y) \hat{i} + (a_z b_x - a_x b_z) \hat{j} + (a_x b_y - a_y b_x) \hat{k}$$

Regola mnemonica: (regola di Sarrus) le componenti cartesiane del prodotto vettoriale si ottengono con lo sviluppo del determinante

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$a_y b_z \hat{i} + a_z b_x \hat{j} + a_x b_y \hat{k} - a_z b_y \hat{i} - a_x b_z \hat{j} - a_y b_x \hat{k}$$

$$= (a_y b_z - a_z b_y) \hat{i} + (a_z b_x - a_x b_z) \hat{j} + (a_x b_y - a_y b_x) \hat{k}$$

in questo caso

$$a_x = 1.8 \quad a_z = 5.4 \quad a_y = 3$$

$$b_x = 1 \quad b_y = 4.2 \quad b_z = 3.5$$

$$\Rightarrow \vec{c} = 6.3 \hat{i} - 3.3 \hat{j} + 2.16 \hat{k}$$

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