

Soluzione LA 1

$$\text{a) } \gamma \frac{mM_T}{R_1^2} = m \frac{v_1^2}{R_1} \quad \rightarrow \quad v_1 = \left(\gamma \frac{M_T}{R_1} \right)^{\frac{1}{2}} \quad \rightarrow \quad T_1 = \frac{1}{2} \frac{\gamma M_T m}{R_1}$$

$$\text{b) } T_1 + V_1 = \frac{\gamma M_T m}{2R_1} - \frac{\gamma M_T m}{R_1} = -\frac{\gamma M_T m}{2R_1}$$

$$\begin{aligned} \text{c) } \Delta T &= \frac{1}{16} \gamma \frac{M_T m}{R_1} \rightarrow T_1 + V_1 - \Delta T = T_2 + V_2 \rightarrow -\frac{\gamma M_T m}{2R_1} - \frac{1}{16} \gamma \frac{M_T m}{R_1} = -\frac{1}{2} \frac{\gamma M_T m}{R_2} \\ &\rightarrow R_2 = \frac{8}{9} R_1 \end{aligned}$$

$$\text{Qd) } V(x, y, z) = -\alpha xyz(x^2 y + xz^2 + y^2 z)$$

Soluzione LA 2:

$$\text{a) } \gamma \frac{mM_T}{(2R_T)^2} = m \frac{v^2}{2R_T} \rightarrow v = \sqrt{\gamma \frac{M_T}{2R_T}}$$

$$\text{b) } \frac{1}{2}mv_T^2 - \frac{\gamma M_T m}{R_T} = 0 \rightarrow v_T = \sqrt{\frac{2\gamma M_T}{R_T}}$$

$$\text{Qd) } V(x, y, z) = -\alpha xyz(x^3 + y^3 + z^3)$$

Soluzioni LA 3

$$\text{a)} \quad \gamma \frac{mM_T}{R_1^2} = \frac{mv_1^2}{R_1} \rightarrow v_1 = \sqrt{\frac{\gamma M_T}{R_1}} \rightarrow T = \frac{2\pi R_1}{v_1} = 2\pi \sqrt{\frac{R_1^3}{\gamma M_T}} \rightarrow T^2 \propto R_1^3$$

$$\begin{aligned} \text{b)} \quad L &= \frac{1}{2}mv_2^2 - \frac{\gamma mM_T}{R_2} - \frac{1}{2}mv_1^2 + \frac{\gamma mM_T}{R_1} \\ v_2 &= \sqrt{\frac{\gamma M_T}{R_2}} \quad L = \frac{1}{2}\gamma M_T m \left(\frac{R_2 - R_1}{R_1 R_2} \right) \end{aligned}$$

$$\text{Qd)} \quad V(x, y, z) = -\alpha x^2 yz(x^2 + y^2 + z^2)$$

Soluzioni LA 4

$$\text{a) } T_A = \frac{1}{2}mv_A^2, T_P = \frac{1}{2}mv_P^2 \rightarrow v_P^2 = 4v_A^2 \rightarrow v_P = 2v_A$$

$$\text{b) } mv_A r_A = mv_P r_P \rightarrow r_A = 2r_P$$

$$\begin{aligned} \text{c) } E = \text{costante} &\rightarrow \frac{1}{2}mv_P^2 - \frac{\gamma mM_T}{r_P} = \frac{1}{2}mv_A^2 - \frac{\gamma mM_T}{r_A} \\ &\rightarrow \frac{1}{2}mv_P^2 - \frac{1}{2}mv_A^2 = \gamma mM_T \left(\frac{1}{r_P} - \frac{1}{r_A} \right) \rightarrow \frac{3}{2}mv_A^2 = \gamma mM_T \left(\frac{r_A - r_P}{r_P r_A} \right) \rightarrow v_A^2 = \frac{2}{3} \frac{\gamma M_T}{r_A} \end{aligned}$$

$$\text{Qd) } V(x, y, z) = -\alpha xyz(x + y + z)$$