Dato il vettore posizione $\vec{r}(t) = 2cos\left(4t\right)\hat{i} + 2sen\left(4t\right)\hat{j} + (t-3)\hat{k}$

calcolare le espressioni dei vettori tangente, normale e binormale

ed il raggio di curvatura della traiettoria al generico tempo t

- velocita' ed accelerazione

$$\vec{\mathrm{v}} = \frac{d\vec{r}}{dt} = -8\,sen\left(4t\right)\hat{i} + 8cos\left(4t\right)\hat{j} + \hat{k}$$

$$\vec{a} = \frac{d\vec{\mathrm{v}}}{dt} = -32\,cos\left(4t\right)\hat{i} - 32sen\left(4t\right)\hat{j}$$

Versore tangente $\hat{t}$

$$\hat{t} = \frac{\vec{v}}{|\vec{v}|}$$

se $\vec{v} = -8\,sen(4t)\,\hat{i} + 8cos(4t)\,\hat{j} + \hat{k}$

$$|\vec{v}| = \sqrt{64\,sen^2(4t) + 64\,cos^2(4t) + 1} = \sqrt{64\left(sen^2(4t) + cos^2(4t)\right) + 1}$$

$|\vec{v}| = \sqrt{65}$ $\Rightarrow$ $\hat{t} = \frac{1}{\sqrt{65}}\left(-8\,sen(4t)\,\hat{i} + 8cos(4t)\,\hat{j} + \hat{k}\right)$

- accelerazione tangenziale e centripeta

$$\vec{a} = a_t\,\hat{t} + a_c\hat{u}_c$$

$$\left|\vec{a}_t\right| = \vec{a}\cdot\hat{t} =$$

$$= \left(-32\cos\left(4t\right)\hat{i} - 32sen\left(4t\right)\hat{j}\right)\cdot\left(\frac{1}{\sqrt{65}}\left(-8\,sen\left(4t\right)\hat{i} + 8\cos\left(4t\right)\hat{j} + \hat{k}\right)\right)$$

$$= \frac{32}{\sqrt{65}}\left(+8\,sen\left(4t\right)\cos\left(4t\right) - 8\,sen\left(4t\right)\cos\left(4t\right)\right) = 0$$

$$\left|\vec{a}_t\right| = \vec{a}\cdot\hat{t} = 0$$ → l'accelerazione e' solo centripeta

⇨ $$\vec{a}_c \equiv \vec{a} = -32\cos\left(4t\right)\hat{i} - 32sen\left(4t\right)\hat{j}$$

Versore centripeto (normale) $\hat{u}_c$

$$\left|\vec{a}_c\right| = \sqrt{32^2 sen^2\left(4t\right) + 32^2 \cos^2\left(4t\right)} \Rightarrow \left|\vec{a}_c\right| = 32$$

$$\hat{u}_c = \frac{\vec{a}_c}{\left|\vec{a}_c\right|} = \frac{-32\cos\left(4t\right)\hat{i} - 32 sen\left(4t\right)\hat{j}}{32}$$

$$\hat{u}_c = -\cos\left(4t\right)\hat{i} - sen\left(4t\right)\hat{j}$$

Raggio di curvatura ρ

$$\left|\vec{a}_c\right| = a_c = \frac{\mathrm{v}^2}{\rho} \qquad \Rightarrow \qquad \rho = \frac{\mathrm{v}^2}{\left|\vec{a}_c\right|}$$

$$\left|\vec{\mathrm{v}}\right| = \sqrt{65} \qquad \left|\vec{a}_c\right| = 32 \qquad \Rightarrow \qquad \rho = \frac{65}{32}$$

Versore binormale) $\hat{u}_b$

$$\hat{b} = \hat{t} \times u_c =$$

$$= \frac{1}{\sqrt{65}} \left(-8\,sen(4t)\,\hat{i} + 8cos(4t)\,\hat{j} + \hat{k} \right) \times \left(-cos(4t)\,\hat{i} - sen(4t)\,\hat{j} \right)$$

$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

$$\left(\hat{t} \times u_c \right)_x = \frac{1}{\sqrt{65}} \left[8cos(4t) \cdot 0 - 1 \cdot \left(-sen(4t) \right) \right] \hat{i} = -\frac{sen(4t)}{\sqrt{65}} \hat{i}$$

$$\left(\hat{t} \times u_c \right)_y = \frac{1}{\sqrt{65}} \left[1 \cdot \left(-cos(4t) \right) - \left(-8\,sen(4t) \right) \cdot 0 \right] \hat{j} = -\frac{cos(4t)}{\sqrt{65}} \hat{j}$$

$$\left(\hat{t} \times u_c \right)_z = \frac{1}{\sqrt{65}} \left[\left(-8sen(4t) \left(-sen(4t) \right) \right) - \left(8cos(4t) \left(-cos(4t) \right) \right) \right] \hat{k} = +\frac{8}{\sqrt{65}} \hat{k}$$

$$\hat{b} = \frac{1}{\sqrt{65}}\left[-sen\left(4t\right)\hat{i} \ - \ cos\left(4t\right)\hat{j} \ + \ 8\hat{k} \ \right]$$

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